RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2017
THIRD YEAR [BATCH 2015-18]
PHYSICS (Honours)
Paper : V [Gr. A&B]

Date : 15/12/2017
Time : 11.00 am – 1.00 pm
Full Marks : 50

(Use a separate Answer book for each group)

Group – A

Answer **any three** from the following: 3 × 10

1. a) The point of suspension of a simple pendulum executing motion in the x–z plane, oscillates simple harmonically along the x-axis with amplitude a and frequency ω_0.
   i) Obtain the Lagrangian, using suitable generalised coordinate.
   ii) Deduce the equation of motion.
   iii) For small oscillations, show that the equation of motion is that of an oscillator with periodic forcing. 2+2+1

   b) The Lagrangian L is given by $L=\frac{1}{2}A\left(\dot{\theta}^2+\dot{\phi}^2\sin^2\theta\right)+\frac{1}{2}B\left(\dot{\psi}+\dot{\phi}\cos\theta\right)^2$, A and B being constants. Find out the constants of motion of the system. 3

   c) The Lagrangian $L=\frac{1}{2}e^{-kt}\left(\dot{x}^2-\mu^2x^2\right)$. Find out the Hamiltonian H. Is it answered? (k,μ are constants). 2

2. a) i) Show that if the Lagrangian, L, of a system is not an explicit function of time, the quantity $J\equiv\sum\dot{q}_\alpha\frac{\partial L}{\partial\dot{q}_\alpha}-L$, is a constant of motion. (q_α's are the generalised coordinates). 2
   ii) Given the Lagrangian $L(\vec{r},\vec{v})=\vec{r}^{\,2}+\vec{r}\cdot\vec{v}+\vec{v}^{\,2}$, $\left(\vec{v}=\frac{d\vec{r}}{dt}\right)$. Are the linear momentum and angular momentum conserved? 4

   b) What is the difference between the Lagrangian and the Hamiltonian descriptions of mechanics? Solve the problem of a simple pendulum by the Hamiltonian equation of motion. 4

3. a) i) State and explain the principle of virtual work for a constrained dynamical system. Use it to deduce D'Alenbert's principle. 4
   ii) Use D'Alenbert's principle to solve the motion of a body sliding down a frictionless fixed inclined plane. 2

   b) Using Jacobi's identity $\left[[\alpha,\beta],H\right]+\left[[\beta,H],\alpha\right]+\left[[H,\alpha],\beta\right]=0$, where $[\alpha,\beta]$ denotes the Poisson Bracket of α and β, prove that the Poisson Bracket of two constants of motion is a constant of motion. 4

4. a) The Lagrangian L is given by $L = \alpha\dot{q}^2 - \beta\cos q$. Find out the positions of stable equilibrium, and the frequency of small oscillation about any one of them. 4

b) The Lagrangian $L = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \frac{1}{2}\left(\omega_1^2 x^2 + \omega_2^2 y^2\right) + \alpha xy$. Find out the normal modes of the problem. 6

5. a) Deduce the number of degrees of freedom of a rigid body consisting of more than two particles. What do these signify physically? 4

b) Deduce Euler's equation of torque-free motion of a rigid body. Find out the constants of motion with reference to a body-fixed set and a space-fixed inertial system. 6

<u>Group – B</u>

Answer **<u>any two</u>** from the following: 2 × 10

6. a) i) Write down the Lorentz transformations, for the space-time coordinates of an event, between two inertial frames S and S', with uniform relative velocity v along the common x-axis.

ii) Show that the square of the space-time interval between two events is Lorentz invariant.

iii) Distinguish between space-like and time-like intervals.

iv) Show that there is no Lorentz transformation which makes two time-like separated events simultaneous in any inertial frame.

v) Show that two space-like separated events can not be made spatially coincident in any inertial frame, by a Lorentz transformation. 5

b) Two clocks are positioned at the ends of a train of length L (as measured in its own frame). They are synchronized in the train frame. The train travels past an observer stationed on the platform at a uniform speed v. Will the clocks appear synchronized for the observer on the platform? If not, which clock moves faster and by how much when observed simultaneously from the platform? 2

c) Two trains A and B each have proper length L and move in the same direction. A's speed is $\frac{4}{5}c$ and B's speed is $\frac{3}{5}c$. How long does it take for A to overtake B as observed by A? By overtaking we mean the time between the front of A passing the back of B and the back of A passing the front of B. What will be the time taken as observed by B? 3

7 a) Pions have half life 1.77×10^{-8} sec. They come out from an accelerator at a velocity $v = 0.99\ c$. At what distance should they drop to half the original number? 2

b) A radioactive nucleus moving at a velocity $c/5$ relative to the laboratory emits a β-particle at a velocity $0.7c$ with respect to the rest frame of the nucleus and in the direction perpendicular to its direction of motion. What is the velocity and direction of the β particle relative to the laboratory? 4

c) Consider a tube filled with water of refractive index n. A beam of light is allowed to pass through it. What will be the speed of light measured from the laboratory when the water is flowing with speed v along the direction of light with respect to the laboratory? Neglect terms of the order of $\left(v/c\right)^2$ and smaller. 4

8. a) Define precisely a four-vector, and show that its norm is invariant under a Lorentz transformation. Hence show that the norm of the velocity 4-vector is equal to $-c^2$ (in the convention $x_4 = ict$). 4

b) i) Write down the 3-component force $\vec{f}$ (i.e. space part of the force 4-vector) acting on a relativistic particle of (rest) mass m, moving with a velocity $\vec{v}$.

ii) Use this expression to calculate the total relativistic energy E of the particle, and hence its relativistic kinetic energy T. 4

c) Find the norm of the momentum 4-vector $\underline{p}$ and hence show that the relativistic energy-momentum relationship for a particle of rest mass m is given by $E^2 = c^2 p^2 + m^2 c^4$ whose p is the magnitude of the 3-vector momentum $\vec{p}$. 2

9. a) Show that the kinetic energy T of a relativistic particle moving with velocity v can be given by $T = \dfrac{\gamma^2}{\gamma + 1} mv^2$, where the symbols have the standard meaning. 2

b) A particle of rest mass m_1 is approaching a particle of rest mass m_2 which is stationary in the laboratory. The speed of approach of m_1 is u. Find the energy of the whole system from the centre of mass. 4

c) From the transformation property of a 4-momentum under Lorentz transformation find the frequency of a Doppler shifted light. Find the longitudinal Doppler shifted frequency for an observer approaching a source emitting a light of frequency ν. What would be the Doppler shift if the source moves towards the observer? Do you expect the same result for Doppler shifted frequency of sound? If not, why? 4

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